Aim and focus of the NWO/FOM Programme

DYNAMICS OF PATTERNS

1 General aim of the programme

The aim of the joint NWO-FOM program *'Dynamics of Patterns'* is to bring together researchers from various subfields of the sciences whose work focuses on the study and analysis of intrinsically nonlinear spatio-temporal patterns. The goal of this programme is to stimulate research and collaborations in this area through allocation of research grants, through the organization of meetings, by stimulating the sharing of visitors and mutual information, and through actively promoting the field at universities and research institutes.

2 Perspective, background and vision

2.1 The scientific challenge

For a long time, progress in some of the hard sciences was crucially based on stripping down a complex system into its building blocks and by subsequently studying these building elements in isolation. This approach has been enormously successful, both from a purely scientific as well as from a technological point of view — to this line of approach we owe such wide-ranging advances like our present day understanding of the fundamental forces or of the (astro)chemical reactions in the gas phase, the spectacular rise of molecular biology, the mathematical theory of dynamical systems, as well as the emergence of the semiconductor industry and the invention of new materials and new medical instrumentation.

Nevertheless, in the last two decades or so, researchers have become increasingly aware that there are intrinsic limitations to this approach too: complex systems exhibit phenomena that cannot be understood by the classical reductionist approach. We will give some examples below. Simply put, these are systems whose overall dynamics is inherently due to the mutual interaction or competition of its building blocks ("more is different"). Since nontrivial new effects can only arise if the coupling of the various elements is nonlinear, and since most interactions in the physical world are local, the common denominator of such studies is that they involve the emergence of spatially extended patterns as a result of nonlinear interactions and spatial coupling.

These international developments have brought together researchers from the traditionally reductionist fields (physics, chemistry, mathematics) with those from fields where it is very difficult or impossible to isolate a particular aspect of the system experimentally or theoretically (astronomy, meteorology, geophysics and parts of biology) or from fields where there is a technological drive towards inherently complex systems (computer science, engineering of complex fluids). For example, many basic features of biological pattern formation have their origin in reaction-diffusion type systems that are amenable to controlled experimentation. Through a global analysis of such systems, mathematicians have in recent years obtained insight in their bifurcation structure and in many of the fundamental dynamical processes. Likewise, sand ripple and sand bank formation found in the natural environment on sea beds or near the coast nowadays have their counterparts in physics laboratories and stimulate the development of mathematical methods. Other examples abound: the incredible sensitivity of the human ear was recently found to be associated with the fact that the ear operates in the nonlinear regime near a Hopf transition, experiments of unprecedented accuracy open up the possibility to investigate the fundamental links between mixing and chaotic advection, theoretical and experimental work on pinching of droplets uncover new scaling behavior in this common phenomenon and are yielding new ways to explore the rheology of complex fluids, insight into the spontaneous formation of patterns are exploited to control patterning of polymer films, and instabilities that plague chemical engineers working with viscoelastic flows or plasma physicists studying discharges are becoming amenable to analysis which suggests them to be due to nonlinear flow or interfacial instabilities. These insights have all become possible because in the last decade(s) so many new theoretical and experimental tools have become available that we can now delve below the surface of these complex systems.

These observations not only illustrate the challenges posed by nonlinear spatio-temporally extended systems, but also the importance of stimulating cross-fertilization and collaborations between scientific subfields. Cutting edge research on such dynamical processes – 'dynamics of patterns', for short – usually does not fit in with the traditional notions of the role of physicists or mathematicians: for studying phenomena which inherently are due to the competition between various effects or building blocks, researchers constantly face the question which elements are crucial and which are not. In order to make real progress with laboratory experiments on problems that spontaneously occur in nature around us, the solution of an experimentalist's search for a good model system is more often solved by a clever combination of elements or by exploiting analogies with systems or methods studied in a different context, rather than by going e.g. to the purest system, the lowest temperature, etcetera. To do so, a good insight into the theoretical issue or an active collaboration between theory and experiment is often necessary. Researchers who focus on developing a theoretical understanding of such phenomena, face the same dilemma: the real challenge is often more to develop a proper model or a realistic (perturbative) understanding of its behavior than an exact treatment of some limiting toy model. For the mathematician, many new problems are brought in the realm of applied mathematical analysis but progress is not simply a matter of "applying mathematics" — for mathematicians and theoretical physicists alike, it requires to combine analytical power with insight, intuition, judging of data and feedback from collaborators. These elements will be of crucial importance especially in the future, where the challenge will be to bring the newly acquired theoretical and experimental tools to bear on the many complex systems whose understanding has so far remained out of reach.

2.2 The Dutch research landscape in comparison with that elsewhere

With the programme we envision, we aim to promote research in the area of dynamics of patterns which shows the above interdisciplinary characteristics. Why is there in our country a need for a special focused programme to meet this challenge? The reason is that while the presence of excellent groups in several subareas provides a fertile ground, there are various traditional obstacles to the Netherlands playing a leading role in this area.

Within the physics community in our country, what we will for lack of a better word refer to as "phenomenological physics" is mostly found at the universities of technology. At theoretical physics institutes of our general universities, there is hardly any research on phenomenological physics; likewise, experimental phenomenological physics at general universities is very limited, and the course curriculum at general universities hardly includes phenomenological physics at all. As a result of this division, the spectrum of research in this area is more on the applied side than in most other countries; there is no natural base for the type of research we aim to promote, i.e., the research made possible by a combination of advanced theoretical, mathematical and experimental techniques and insight from various fields, and which like the examples mentioned earlier is publishable in high impact journals like Nature and Physical Review Letters.

Certainly at the general universities, mathematicians in our country have traditionally also shied away from this type of applied mathematics. While it is perfectly normal to find a real experiment in the basement of the applied mathematics department at MIT or in Cambridge, this is an unknown phenomenon here. While there are strong communities in the areas of partial differential equations and dynamical systems, few mathematicians in our country are actively involved in the process of model building or in collaborations where identifying the essential ingredients is more important than a mathematical proof of the behavior of a well-defined model.

In comparison with the situation in other countries, this situation *is* somewhat of an anomaly. In the Anglo-Saxon world and, to some extent, in France, it is very common to find top researchers in this field at the major universities, both within applied mathematics departments and in physics departments. It is useful to give some explicit examples. We have in mind the contributions of Kadanoff (physics, Chicago) to complex systems, turbulence, granular media as well as the pinching of droplets, those of Holmes (mathematics, Princeton) on dynamical systems as well as turbulence and bio-locomotion, those of Newell (mathematics, Tucson and Warwick) whose contributions range from pure mathematics to the understanding of pattern formation in fluids as well as in nonlinear optics, those of Goldstein (physics, Tucson) to the mathematics of free twist and bend of strings which as shown below apply to the dynamics of bacterial flagella, the contributions of Sandstede (mathematics, Ohio) and Proctor (applied mathematics, Cambridge) to opening up many fundamental topics in the theory of pattern formation to global mathematical analysis, the experimental work of the separate groups of Bonn, Couder, Fauve and Tabeling (physics, ENS Paris) on viscous fingering, crystal growth, granular media and fluid dynamics, the work of Keener (mathematics, Utah) to physiology and biophysics, the contributions of Nozières (ILL Grenoble and Collège de France) to plastic friction, those of Langer (ITP, Santa Barbara) to dendritic growth, fracture and earthquakes, the experimental work of Fineberg (Weizmann Institute) on fracture, of Gollub (physics, Haverford) on growth phenomena, mixing and granular media, the contributions of Kaper (mathematics, Boston) to the understanding of bacterial growth phenomena and chemical wave experiments, of Eggers (applied mathematics Bristol/Ann Arbor) on singularities and scaling in fluid flows and materials science, the contributions of Jones (mathematics, Brown and North Carolina) who is playing a leading role both in pure mathematics as well as in oceanography, and those of Brenner (mathematics, MIT) to sonoluminescense, the formation of bacterial patterns as well as many other pattern forming instabilities.

3 Objectives and focus of the programme

As explained above, the long-term goal of the programme will be to build and strengthen a community in applied nonlinear science and increase its impact. This is done directly through stimulating and funding new collaborative projects in the areas identified below, and through organizing meetings like those organized by Lohse and Doelman in Twente in 2000 and 2002. In addition, we aim to actively use the program as a vehicle to stimulate research and appointments in this area, both through contacts with individual departments and institutes and, in exceptional cases, through support for hiring new faculty members and for start-up funds.

The main research focus of the programme will be to contribute to the fundamental understanding of specific nonlinear dynamical phenomena of spatially extended systems. These should be systems of broader interest within the sciences whose dynamics is inherently complex and based on the competition of various effects. The programme includes theoretical as well as experimental research aimed at a fundamental understanding of the phenomena.

Within the general field of "dynamics of patterns" there are two main methodologies under whose heading various recent developments come together in such a way that significant novel insights can be expected to occur soon. These are:

– Building blocks of spatio-temporal dynamics and their interactions.

– Instabilities in complex systems.

Loosely speaking, with building blocks of spatio-temporal behavior we mean localized solutions (pulses, defects, fronts or interfaces) which organize most of the dynamics of the system. Typically, in such cases the state of the system away from the building blocks is relatively trivial, but the interaction between them, which is mediated by the bulk states (e.g. through diffusion fields) is crucial. The second theme, instabilities in complex systems refers to the other situation in which systems show a bulk instability, so that their dynamics does not naturally split up into the motion of localized solutions like interfaces.

The content of the programme is made concrete below by taking two cross-sections through it, first a topical one, then a methodological one according to the themes identified above.

3.1 Cross-section: phenomena

We identify the following main research topics where important progress is likely to be made in the coming years, and where science within the Netherlands has the potential to become a major player:

- **Nonlinear dynamics and growth of free surfaces and lines**
  In the last decade, the nonlinear interfacial growth phenomena like dendritic growth or viscous fingering have become so much better understood that the ingredients are available to turn to growth processes in complex systems. Examples of new projects in this direction abound: *(i)* viscous fingering allows one to unravel the specific phenomena (shear thinning, influence of surfactants) of associated with a complex fluid;[15] *(ii)* the specific differences and similarities of the growth of bacterial colonies and physical growth phenomena are becoming studied;[16] *(iii)* the first detailed nonlinear growth model has recently become proposed[17] for the growth of filamentary micro-organisms like those of Fig. 1*(a), (b)* (the challenge is the incorporation of both bio-elasticity of the growing membrane and diffusional feeding through the outer region for an inherently strongly curved membrane whose shape is not known a priori); *(iv)* nonlinear dynamics techniques have recently been used to exploit the similarities of a propagating discharge front with a growing interface, including the spontaneous occurrence of tip-splitting;[18,19] *(v)* the nonlinear dynamics during necking and pinching of a droplet has become amenable to theoretical analysis and experimental study and shows remarkable scaling behavior; suggestions are that this may yield a new tool to probe complex fluids;[10] *(vi)* when vortices penetrate a superconductor, they often do so as a well-defined front reminiscent of an interface.[20] Techniques developed for analyzing the nonlinear dynamics of moving interfaces surely apply in this case, but this area remains to be explored; *(vii)* a spectacular example of the nonlinear dynamics of a string-like object in three-dimensional space is shown in Fig. 1*(b,c)*, which illustrates the nonlinear dynamics of a kink in the handedness of a bacterial flagellum induced by an external flow. The theoretical challenge for understanding such dynamical problems resides in the formulation of realistic dynamical equations for a free curved one-dimensional object embedded in three dimensions.[21]

- **Instabilities and nonlinear dynamics**
  In the last two decades, many new experimental and theoretical tools have been developed to study relatively simple bulk pattern dynamics in real systems, such as the formation of convection patterns in ordinary fluids in a temperature gradient, chemical reaction patterns, etcetera. These tools give the opportunity to tackle new more complex systems which exhibit intrinsically nonlinear behavior: *(i)* the polymer industry has been plagued since long by the occurrence of irregularities at the surface of a polymer flowing out of a tube, see Fig. 2*(a)*. Only this year have techniques developed in the area of pattern formation been able to identify a visco-elastic flow instability as an intrinsic route towards such unwanted behavior.[22] The work even opens up avenues to tackle visco-elastic turbulence,[23,24] turbulence of polymer melts in the zero Reynolds number limit. This is just one example — complex fluids exhibit a plethora of instabilities (e.g. shear-banding in a granular medium, instabilities of yield-stress fluids, nonlinear drainage of foams) which remain to be unraveled; *(ii)* the key ingredients that can lead to dynamical self-replication — an issue of relevance to morphogenesis — in reaction-diffusion type models begin to be identified;[25,26] *(iii)* recent research has opened up the possibility to understand proteine dynamics inside a bacterium in terms of the nonlinear

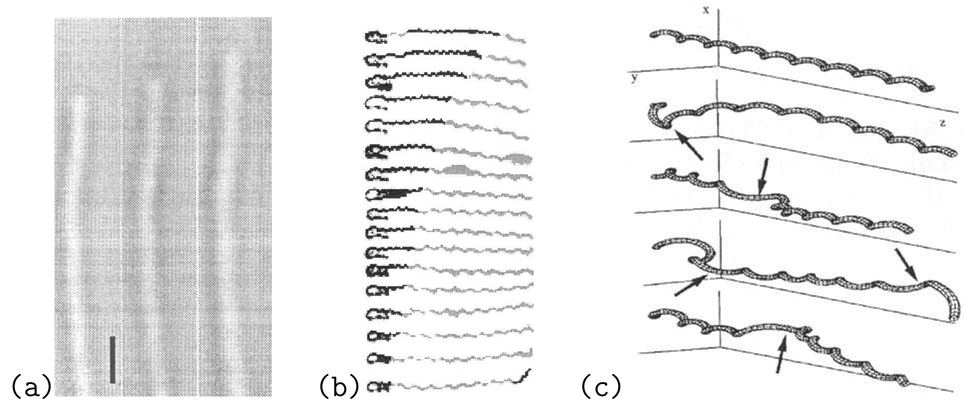

Figure 1: *(a)* Three successive photos[17] of the filamentary micro-organism *Streptomyces coelicolor A3(2)* during growth, with the bar indicating $1\mu m$. The growth takes place only at the strongly curved tip. A recently developed interface model[17] for this organism captures the growth dynamics quite well, and mathematically exhibits self-similar growth dynamics. *(b)* Chirality transformation in a *salmonella* flagellum induced by imposing a flow past the bacterium. Time increases from bottom to top, and grey denotes the lefthanded normal chirality and black the righthanded curly state. Note the propagation of the kink-like transition region down the flagellum.[21] *(c)* Numerical solution of the nonlinear model for the motion of the flagellum in a situation comparable to the experiment. The arrows denote the regions where the handedness changes.[21]

dynamics: the proteine dynamics that allows a bacterium to identify the center of the cell where it should split, is traced to a Turing-type reaction diffusion type instability.[28] This research is another good illustration of our general remarks: from a purely formal point of view, the nonlinear dynamics is relatively trivial once the model has been formulated — the challenge lies in combining the global insight into nonlinear bulk instabilities with information from biology about the binding and diffusion of the polymers. Many applications of this line of research lie ahead; *(iv)* While the essential dynamics responsible for the characteristic features of coffee stains was recently identified,[29] related apparently simple problems like the unusual scaling behavior of the radius of evaporating droplets remain a mystery, even though it is believed that the essence is hidden in the nonlinear scaling behavior of the thin film equations; *(v)* large scale eddies in the ocean are coherent structures that are generated by instabilities of an underlying large scale flow pattern.[6] These instabilities have a truly complex, and thus only partly understood, nature due to the nonnormal character of the governing equations;[31] *(vi)* new analytical techniques and stability analysis applied to dynamic fracture phenomena have uncovered that nonlinear dynamics plays a prominent role in some of the behavior, e.g. in splitting of the fracture tip.[11]

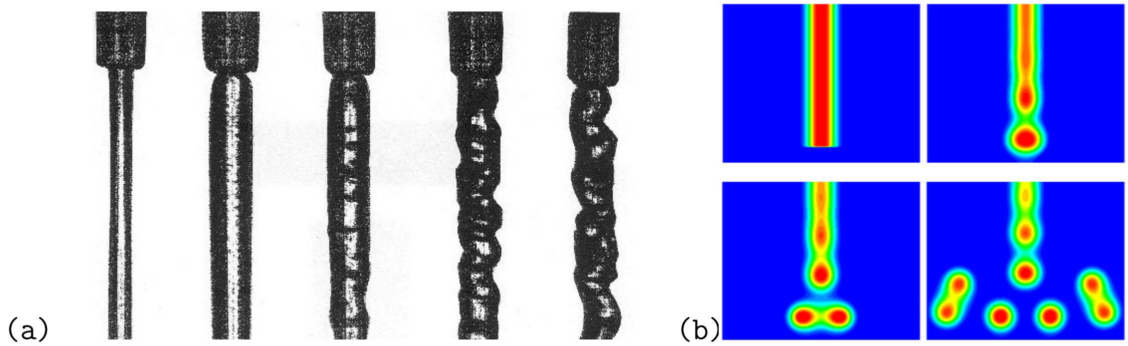

Figure 2: *(a)* Five snapshots of polymer solution flowing out of a tube, with overall flow rate increasing from left to right. At low flow rates the outflow is smooth, beyond some threshold the surface becomes irregular. A recent nonlinear amplitude expansion shows that this results from a nonlinear (subcritical) viscoelastic instability of the Poiseuille flow in the tube.[22] It has been recently been suggested[24] that cases like the rightmost picture, the viscoelastic flow is turbulent, even though the Reynolds number is practically zero. The structure of the expansion results[22] suggests new classes of amplitude equations for these regimes. *(b)* Interaction of building blocks: a perturbed localized 'stripe' bifurcates as time evolves into self-replicating 'spots', in simulations of a biological model for "morphogenesis".[27] The study of this self-replication process has recently given rise to various new mathematical developments.[7,25,26]

3.2 Cross-section: methodology

Almost all of the phenomena we have sketched above touch on or give rise to issues in the field of partial differential equations. In fact, within mathematics, 'Dynamics of Patterns' corresponds to the analysis of the dynamics of solutions of nonlinear partial differential equations of evolution type, i.e. the behavior of infinite dimensional dynamical systems. This field is nowadays evolving rapidly, especially through the interaction between the classical theory of partial differential equations and the more geometrical approach of the theory of dynamical systems. If one takes cross-section of the program along the methodology axis one invariably encounters mathematical challenges that are essential for the further development of the theory of infinite-dimensional nonlinear systems:

- **Building blocks and their interactions**
  The localized structures that we have mentioned form in more technical terms *coherent structure solutions* of the governing dynamical equations: the building blocks are usually uniformly translating solutions (depending on space and time only in the combination $x - vt$) which have a well-defined spatial extent. Such solutions can often be studied in detail using the underlying low-dimensional dynamical system of ordinary differential equations that describes them,[1] and they can take forms ranging from a localized traveling pulse through a nerve fiber to a vortex or a front. In systems whose dynamics is dominated by such building blocks, the challenge lies first in identifying these structures (often a nontrivial task) and secondly in developing a good description of their interaction and mutual dynamics.[1,32]

Neither the mathematical nor the physical theory for "perfect" building blocks — the solitary localized patterns in a homogeneous background state — is completely developed. Nevertheless, the state-of-the-art has recently grown to such a high level that breakthrough results concerning the understanding of fully dynamical patterns governed by motion of interacting coherent structures governed by ordinary differential equations like fronts or other localized structures is within reach. Moreover, a very recent mathematical advance[33] on classifying coherent structures in oscillatory systems opens up new avenues for studying coherent structures in such systems. Within the context of interacting coherent structures we distinguish here the following sub-areas.

(i) Transient dynamics. Coherent structures are often not stable in the classical sense (i.e. for all time): they may appear, exist for some (long) time, and eventually disappear again. This continuous transient process of emergence, slow evolution followed by annihilation is remarkably subtle, but various aspects can nowadays be studied due to the recent developments in the field of localized patterns.[4,35–37] A key role is played by the so-called self-replicating patterns[25,26] shown in Fig. 2*(b)*: especially for these patterns there is an intimate relationship between experimental observations, computer simulations, modeling and analysis.[9,26,34,36]

(ii) Defects. Defects are natural structures that have been observed and studied in many conserved near-equilibrium systems. However, many new fundamental questions about interactions between defects, or between defects and the background states pose challenging and relevant problems. Relatively simple experiments[38,39] exhibit phenomena that can yet not be explained by models or analysis. For instance, the most important "tool" to understand defects, the phase-diffusion equation, has been successfully applied to many systems,[12,40] but there is no theoretical explanation of the character of traveling defects (as observed at certain species of fish[38]). Moreover, the theoretical understanding of the phase-diffusion equation itself is also still far from complete, but significant progress is (almost) within grasp.[41]

(iii) Evolution of filaments and interfaces. As the earlier examples illustrate, a coherent structure may have the character of a curve or a surface in two or three (spatial) dimensions. The evolution of these interfaces is usually driven by intrinsic features (e.g. elasticity, "resistance" against bending) and by its interaction with the ambient space. If so, mathematical techniques are available to map the internal dynamics on certain reduced, but often highly complex, effective interface models.[1,42,43] These models in turn are closely related to phenomena studied in the context of the geometry and evolution of curves and surfaces.[5,44] Such effective interface models or moving boundary models display complicated nonlinear dynamical behavior; this dynamics is often not understood in detail, but a sufficient amount is known about the various dynamical regimes that the gross behavior of specific systems can often be mapped out in first instance.

- **Instabilities in complex systems**
  The concept of instability lies at the center of our understanding of pattern formation.[3,12] Initiated by the pioneering work of Landau and Turing, and

stimulated by experimental observations, a weakly nonlinear theory that governs the dynamics of instabilities has been developed in the last quarter of the 20th century.[1,2,12] This Ginzburg-Landau theory forms a cornerstone of the field pattern formation. A remarkably rich range of phenomena can be analyzed, understood and predicted by this theory: the onset of convection in a fluid or in the atmosphere, the formation of hexagonal structures in liquid crystals or polymer films, the dynamics of sand banks in seas or rivers, to mention but a few. Even though the applicability of this Ginzburg-Landau formalism is still mostly restricted to describing instabilities of the most simple (or 'generic') kind and close to threshold, the state-of-the-art of the field is such that various of the challenging complex problems mentioned earlier can be tackled.

(i) Nonlinear subcritical instability. Most of the present understanding of the evolution of instabilities has its origin in linear mechanisms. However, it is known that in various situations, there is no linear instability. The most well-known classical example is the problem of shear flow instability,[22,45] and according to the recent work mentioned above polymer flow is a new low Reynolds number variant of this. Moreover, even when there is a linear destabilization, a system may exhibit pattern formation (long) before the critical threshold is reached, due to non-normal effects.[46] This is truly an emerging field with relevance that ranges from engineering applications to the foundations of the theory of pattern formation.

(ii) Interacting instabilities. Many patterns observed in nature are formed by interacting instability mechanisms. This is often due to an inherent symmetry or a conservation law in the system.[47] The dynamics of these patterns are intricate, and sometimes even counter-intuitive. For instance, due to these interactions structures can be observed, that are unstable, and thus invisible, in the reduced Ginzburg-Landau setting.

(iii) Instabilities of nonhomogeneous states. The classical theory of instabilities considers the destabilization of a homogeneous state. A significant new challenge is understanding more general situations in which nontrivial patterns are destabilized.[40,41,48,49] This topic is closely related to the above mentioned analysis of defects.

4 Relation with other programs

As we have indicated, some of the challenging problems for the research the program wishes to promote come from biology. The life sciences are a vast subject, for which various stimulation programs are available. Because the program definitely want to take advantage of the challenges and research topics this field offers, and contribute to their solution, this program is kept open to the small subset of biophysical problems which are related to the main themes that identified above, and for which advanced theoretical modeling of nonlinear growth of curved lines and interfaces or of instabilities is indispensable. Moreover, experience shows that these topics, which require highly sophisticated mathematical modeling, do not find a natural base within other programmes.

While some aspects touch on programmes in fluid dynamics, in practice the distinction between those programmes and this one is quite clear: full scale Navier-Stokes modeling naturally falls into the fluid dynamics programmes — this program has a different, almost complementary, emphasis. Where it touches on fluid dynamics, it is more natural for problems related to the major themes which are analyzed in terms of reduced descriptions (thin film equations, Ginzburg-Landau amplitude equations, etcetera). While some of the topics studied within this program may have some indirect bearing on flow instabilities in granular media, as a rule granular media research should be funded through the granular media programme of FOM.

The computer is, of course, an invaluable tool for many aspects of the research described here – numerical simulations are essential for guidance, challenge and stimulation of the work within this programme. Nevertheless, the focus of this program is very different from computational science programs. In those, computations play the central role. Since gaining analytical insight and understanding in the dynamics of patterns is the central focus of this programme, the computer will not at all play a central role. In this sense, the present program is complementary to computational ones.

The first 14 reference concern recent general articles and books that are relevant to this proposal. The references following those are cited references.

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